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REVIEW

Wave Interactions and Fluid Flows. By A. D. D. CRAIK. Cambridge University Press, 1985. 322 pp. £35.

Wave motion is a universal feature of many physical systems. Throughout the last century and for much of this attention was largely centred on the linear theory which has now been developed to a high degree of sophistication. However, apart from developments associated with the pioneering work of Stokes on water waves and Riemann on acoustic waves, there was comparatively little interest in nonlinear waves in fluids until the early 1960s, when there was a rapid growth in the study of wave interactions, nonlinear stability, solitons and other aspects deriving from nonlinearity, which has continued unabated until the present time. Notable in the vast literature are the monographs by Whitham (1974), Joseph (1976) and Phillips (1977), the collection of articles by Swinney & Gollub (1981), as well as the various texts on solitons and the inverse scattering transform which have appeared in the last few years. Of course, the universality of wave motion ensures that similar developments took place in other physical contexts, and in particular some aspects of the theory of nonlinear waves in fluids were anticipated in solid-state physics, nonlinear optics and plasma physics. This monograph by Craik is timely in that it fills a gap in the existing literature by focusing on wave interactions, with the main emphasis on surface waves, internal waves and shear flows. Undoubtedly the principal value of this text lies in its account of the state-of-the-art of weakly nonlinear wave interactions in shear flows, and the inextricably related topic of weakly nonlinear stability, written by one who has been involved in these subjects since their inception in the early 1960s. The discussion largely centres around the nonlinear evolution equations which result from an appropriate weakly nonlinear theory, and a welcome feature is the emphasis given to the universality of these evolution equations. Of course, it is inevitable that any state-of-the-art account of a subject which is still developing will become dated, and this text will be no exception. Nevertheless it will for many years to come be required reading for all research workers who have a serious interest in nonlinear waves in fluids.

In order to give some idea of the topics covered I shall list the chapter headings. These are (1) Introduction, (2) Linear wave interactions, (3) Introduction to nonlinear theory, (4) Waves and mean flows, (5) Three-wave resonance, (6) Evolution of a nonlinear wave-train, (7) Cubic three-and-four-wave interactions, (8) Strong interactions, local instabilities and turbulence: a postscript. Chapter 2 on linear waves is a necessary precursor to the main topic of weakly nonlinear wave interactions. It is necessarily condensed and the novice to the study of wave motions will find the chapter difficult. On the other hand the reader who has been well primed in classical linear wave theory will find much to enjoy and to provide stimulation. An instance of this is the early introduction in §2.3 of a definition of wave energy which differs from that customarily found in more elementary texts on linear waves. The definition given here is related to the pseudo-energy defined by Andrews & McIntyre (1978*a, b*), and in some simple situations is the ratio of the wave action density to the absolute frequency (i.e. the frequency determined in a fixed frame of reference). In the presence of an underlying shear flow this differs from the wave energy as conventionally defined by the ratio of the absolute frequency to the Doppler-shifted frequency. Of course, it is true that the definition given here is the more fundamental quantity in that it is a wave invariant in the absence of dissipation when the basic state is time-dependent. A related instance is the consequent observation that waves may possess either positive or negative energy, a result which could be puzzling at first sight to a reader whose sole background was classical linear wave theory. I feel that these, and other, concepts would have benefited from the earlier introduction of Lagrangian (or Hamiltonian) principles which would then have provided a unifying framework for the development of the theory. As it is, the reader must wait until Chapter 4 before Lagrangian concepts are first introduced. Another example of this is the demonstration in §2.3 that the coupling of positive and negative energy waves leads to instability. In the text this seems to depend on the fortuitous fact that a certain coupling constant is positive, whereas a derivation from an averaged Lagrangian shows immediately that the coupling constant is necessarily positive. Yet another example occurs in §4.3 in the discussion of the reflection and transmission of internal waves from a vortex sheet which I feel would have benefited from the use of the wave action flux invariant. Again the reader must wait until Chapter 4 (§§11.4 and 11.5) before meeting wave action concepts. Similar comments apply to the discussion in §5.3 on critical layers. On the credit side it must be said that the style of this text, which is to introduce the reader to basic concepts through simple problems, does stimulate the diligent reader to work through these problems, perhaps from a different perspective, and thus gain in understanding.

Chapter 3 gives the reader a brief introduction to the generic evolution equations which are subsequently derived and discussed in various physical contexts in Chapters 5, 6 and 7. Interpolated, in Chapter 4, is an account of wave–mean flow interaction. In the general style employed throughout this text, the reader is first introduced to this topic by examples taken from channel flows, water waves and stratified shear flows, where the wave-induced Eulerian mean flow is calculated using the conventional Reynolds-stress type of argument. Although Craik has always, quite correctly, used the mean momentum equations to calculate the wave-induced mean flow, I felt that, in view of the current confusion in some of the literature over the concept of 'wave momentum', it would have been useful to have included a more extensive discussion of the care that is necessary in calculating the wave-induced mean flow and the necessity to avoid using *ad hoc* 'wave-momentum' arguments. Wave action is introduced in §11.4, although I found it a little confusing here that Craik has reversed the conventional notation by using ω' for the absolute frequency and ω for the intrinsic, or Doppler-shifted frequency, in contrast to the notation used in Chapter 2. The powerful generalized-Lagrangian-mean formulation of Andrews & McIntyre (1978*a*, *b*) is introduced in the middle of the chapter which then concludes with applications to the generation of spatially periodic mean flows in boundary layers, and to the phenomenon of Langmuir circulations in the ocean. The inclusion of experimental results here, and elsewhere throughout the text, is to be welcomed, although I found the discussion of these two important applications to be inevitably too brief. Suffice it to say that the interested reader will be stimulated to search through the original literature.

In Chapter 5 the theory of three-wave resonance is developed, beginning with the familiar triad-interaction equations for the conservative case, and applying the results to capillary–gravity waves and internal waves. A welcome feature here is the discussion on the explosive interaction which occurs when waves of differing energy sign interact with the wave of greatest absolute frequency having energy of different sign from the other two waves. This phenomenon, widely known in plasma physics, has not received the attention it deserves in the fluid-mechanics literature. The discussion then goes on to consider some of the interesting solutions recently obtained for the case when spatial modulations are allowed for in the interaction equations. The chapter concludes with a discussion of wave interactions in shear flows, motivated in part by recent experimental results which have shown evidence of three-wave interactions in boundary layers.

In Chapter 6, which treats the theory of weakly nonlinear wavetrains the discussion naturally centres around the familiar nonlinear Schrödinger equation, and its various extensions, although other evolution equations are briefly discussed. For inviscid fluids the principal application discussed is water waves where the nonlinear Schrödinger equation and its two-dimensional extension have proved to be quite successful models of weakly nonlinear water waves, and in particular provide a simple model of Benjamin–Feir instability. For applications to shear flows the corresponding evolution equations have complex coefficients and solutions are consequently harder to obtain: here agreement with experiments is less convincing due in part to the difficulty that a weakly nonlinear theory cannot adequately cope with the finite growth rates predicted by linear stability theory. The chapter concludes with accounts of numerical work on finite-amplitude water waves, and finite-amplitude effects in shear flows. Inevitably these sections are too brief to be more than tantalizing short introductions to some of the interesting results now available; I felt that they both could have been more usefully included in the following chapter. The very brief discussion on nonlinear critical layers does not do justice to this fascinating topic, and in particular I felt that the important distinction between unstratified and stratified cases was not given sufficient prominence.

The final two chapters deal with higher-order wave interactions. Chapter 7 begins with an account of the Zakharov equation which describes weakly nonlinear wave interactions for water waves, and the improvement over the nonlinear Schrödinger equation model that it provides in describing the instability of small-amplitude water waves. The remainder of this chapter and the final one develop the connection between wave interactions and nonlinear stability theory within the context of Taylor–Couette flow, Rayleigh–Bénard convection and planar shear flows. These are very much state-of-the-art discussions on topics for which recent theoretical, numerical and experimental results are continuing to reveal fascinating and extremely intricate phenomena. The book ends appropriately with speculations about the role of the theory in describing transition to turbulence.

I enjoyed reading this book and can recommend it to all with an interest in waves and stability in fluids. I found very few misprints, and I could identify only a very few significant omissions from the extensive and useful bibliography. I hope that these and other minor faults will be corrected in a forthcoming paperback edition.

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